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ESSAY *on the* BEST METHOD *of* ASCERTAINING *the* AREAS *of* COUNTRIES *of* CONSIDERABLE EXTENT.

By the Rev. JAMES WHITELAW, *Vicar of St. Catherine, Dublin, and M. R. I. A.*

Read April 25, 1795.

MODERN geographers, and particularly thoſe of Germany, have, of late years, cultivated a ſcience to which they have given the name of *Statiſtics*, the object of which is, to aſcertain the extent, population, denſity of population, military and naval force, revenues, national debt, &c. of the various ſtates of Europe. This ſcience, intereſting to every enlightened mind, and peculiarly ſo to the ſtateſman, is, as yet, in its infancy; and that particular branch of it, which enables us to determine the areas or ſuperficial contents of countries, has been hitherto ſo neglected, or conducted on ſuch erroneous principles, that the calculations of different authors are perfectly inconſiſtent with each other, and with truth. Some areas in our tables are, it muſt be acknowledged, ſufficiently accurate; but as truth and error are advanced with equal confidence, and equal authority, ſuch are uſeleſs to the reader, who, incompetent to make a ſelection, cannot proportion his aſſent to their merit.

THE ſtrange inconſiſtencies that occur in ſtatiſtical tables will appear more ſurpriſing, when we conſider that the preſent cultivated ſtate of geography, though far from perfection, leaves little room for thoſe enormous errors which were formerly inevitable: When ſuch, therefore, occur in modern publications, we may juſtly, I think, impute them either to want of ſkill or want of care in the author; and conclude that his method was incorrect, or that he performed the operation on very erroneous maps. The following are a few of the numerous inconſiſtencies to which I allude, but ſufficient to ſhew the very imperfect ſtate of this part of the ſcience of ſtatiſtics.

Europe in ſquare Miles, of 60 *to a Degree, by various Authors.*

		Difference between each and Kitchin's map.
Kitchin's map of Europe, according to Zimmermann, - -	2,180,466—	
Buſching - - - -	2,432,000—251,534	
Statiſtiſche Ueberſicht or German political Tables 1786 - -	2,712,112—531,646	
Templeman's Survey of the Globe -	2,749,314—568,848	
Crome, on the Size and Population of Europe 1785 - - -	2,785,440—604,974	
Bergman's Natural Geography -	2,906,112—725,646	

Here the laſt difference is above four times the area of the kingdom of France.

AMIDST

AMIDST errors of ſuch magnitude how can we diſcover truth? is an obvious queſtion. It has often employed my thoughts, and I take the liberty of ſubmitting the reſult to the Academy.

A *ſmall* portion of the earth's ſpherical ſurface may be accurately repreſented by a map drawn on a plain ſurface, and the menſuration of ſuch map, by the ordinary methods uſed by ſurveyors, will give the area of the country it repreſents, with ſufficient correctneſs: but in maps of extenſive regions the rule no longer holds, and the impoſſibility of juſtly delineating any conſiderable portion of a ſphere on a plain has induced geographers to have recourſe to the various methods of projection, ſuch as the ſtereographic, orthographic, &c. Maps conſtructed on ſuch principles may give the relative ſituation of places, with reſpect to bearing, meridians and parallels, but cannot convey any accurate idea of form or ſuperficial contents: they are, at beſt, but diſtorted reſemblances, and the greater the extent they embrace the greater the diſtortion, and the more incorrect the area: to ſuch maps a ſcale cannot be applied, and they are conſequently totally unfit for the purpoſe of menſuration.

THAT the area of even a ſingle kingdom of moderate dimenſions muſt differ very conſiderably from that of its map, will be evident from conſidering the different methods of conſtructing the latter.

THE uſual kinds of projection for particular maps are the ſtereographic on the plane of the horizon, the conical, and the circular.

IN the firſt, which is founded on the rules of perſpective, equal tracts of country are repreſentéd as very unequal, gradually encreaſing as they recede from the centre of the plane of projection; hence no common ſcale can be applied to the map, which is therefore totally unfit for menſuration.

THE conical projection is ſo called, as it is produced by the application of a cone to a ſphere, according to certain rules. In this the meridians are right lines on the cone, converging at its vertex, and perpendicular to the parallels of the lat. which are circles, deſcribed from the vertex as their common centre: The diſtance between the meridians is correct, only on the two parallels formed by the interſections of the ſphere and cone, between which they will approach too near, and every where elſe recede too far from each other; and hence the area of that part of the map delineated on the portion of the cone which falls within the ſphere will be leſs than the truth, while without thoſe limits it will proportionably exceed it. This is an ingenious and elegant projection, and as the conical area between its extreme meridians and parallels is perfectly equal to that portion of the ſphere it repreſents, if the limits of the country delineated on it perfectly coincided with thoſe lines, it would neceſſarily give the true area: But this never happens, and to

ſpeak

ſpeak with mathematical accuracy it is liable to the ſame objection as the laſt, viz. that a ſcale cannot be applied to it; for countries, however, of moderate extent in latitude, it is the beſt that can be deviſed.

In the circular projection the parallels of lat. are circles, deſcribed from the pole as the common centre, with radii equal to the complements of their reſpective latitudes: the meridians preſerve their due diſtances on the parallels, becoming gradually curvilineal, and cutting the parallels more and more obliquely, as they recede from the central meridian; but as this alone can be a right line, and cut the parallels perpendicularly, it is obvious, that this projection muſt produce diſtortion in the map, and conſequently an error in its area, which, if it is greatly extended in longitude, will at length become enormous: for countries, however, of moderate dimenſions, it is one of the leaſt objectionable projections, and a ſcale may be adapted to it with ſufficient accuracy, though not with geometrical preciſion.

There is another method of projection, uſed by the Sanſons, in which the parallels are right lines, and the meridians, which obſerve their due diſtances on the parallels, are parabolic curves. This is admiſſible only in delineating countries of inconſiderable extent in longitude, and Mr. D'Anville has preferred it for his map of South America: In all other caſes it produces much greater diſtortion than any of the above methods.

In the cylindrical projection the countries on each ſide of the equator are conceived to be delineated on the ſurface of a cylinder, cutting the ſphere in two parallels, nearly midway between the equator and the top and bottom of the map: The meridians and parallels are right-lines, cutting each other at right-angles, and the former obſerve their due diſtance only on the interſections of the cylinder and ſphere: This projection is evidently the beſt for the tropical regions; but, as it is not of general application, and partakes of the perfections and imperfections of the conical, it is ſufficient to have mentioned it.

Although it is thus evident that the area, reſulting from the ſimple menſuration of every ſpecies of map, however projected, muſt, (except the country repreſented be of ſmall extent) differ conſiderably from the true area, yet there is reaſon, I think, to ſuſpect that the generality of ſtatiſtical writers have only performed the operation in the uſual way, and perhaps on maps injudiciouſly ſelected, without taking any precaution to correct the errors which muſt inevitably ariſe from ſuch a proceſs; at leaſt the ſtrange inconſiſtency obſervable in their tables, with their want of candour, in not communicating to the public their method and materials, ſubject them to this ſuſpicion. A few, indeed, have given us ſome numbers ſufficiently correct, but by what means they diſcovered truth, as they are ſilent on the ſubject, we are ignorant. I ſhall, therefore, make no apology for offering the following method of nearly determining the true areas of maps: It is, I believe, new, and will be found, I hope, ſufficiently

ſufficiently accurate for the purpoſes for which ſtatiſtical tables are intended; for minute preciſion is impoſſible, and if poſſible would be unneceſſary.

Method of nearly determining the true Areas of Maps on the Conical and Circular Projections.

Parallels of latitude, diſtant from each other one degree, will divide the ſpherical ſurface of the earth into 180 narrow zones; the area of each of theſe, in ſquare miles of 60 to a degree, is found by multiplying its ſine in miles and decimals of a mile by 21600, the circumference of a great circle in ſuch miles; and the area of each zone, thus found, divided by 360, will give the area of each of the quadrilateral ſpaces, formed by the parallels, which include the zone and two meridians diſtant one degree of longitude from each other: On this principle the following table was conſtructed.

Lat. of the Zones.		Area of Quadrilat.	Lat. of Zones.	Area of Quadrilat.	Lat. of Zones.	Area of Quad.	Lat. of Zones.	Area of Quad.	Lat. of Zones.	Area of Quad.
Between 0	and 1°	3594 . 8	15—16	3462 . 0	30—31	3097 . 5	45—46	2519 . 7	62—63	1660 . 0
1	2	3593 . 7	16—17	3446 . 2	31—32	3065 . 1	46—47	2474 . 6	63—64	1604 . 0
2	3	3591 . 5	17—18	3429 . 2	32—33	3031 . 9	47—48	2428 . 6	64—65	1547 . 7
3	4	3588 . 2	18—19	3410 . 4	33—34	2997 . 8	48—49	2382 . 1	65—66	1490 . 7
4	5	3583 . 8	19—20	3390 . 0	34—35	2962 . 6	49—50	2334 . 7	66—67	1433 . 4
5	6	3578 . 4	20—21	3367 . 3	35—36	2926 . 7	50—51	2286 . 8	67—68	1375 . 5
6	7	3571 . 8	21—22	3344 . 8	36—37	2889 . 8	51—52	2237 . 8	68—69	1317 . 5
7	8	3564 . 2	22—23	3321 . 3	37—38	2852 . 1	52—53	2188 . 4	69—70	1259 . 0
8	9	3555 . 5	23—24	3296 . 7	38—39	2813 . 4	53—54	2138 . 2	70—71	1201 . 2
9	10	3545 . 6	24—25	3271 . 3	39—40	2773 . 9	54—55	2087 . 6	71—72	1141 . 8
10	11	3534 . 8	25—26	3244 . 7	40—41	2733 . 6	55—56	2036 . 1		
11	12	3522 . 8	26—27	3217 . 3	41—42	2692 . 4	56—57	1984 . 2		
12	13	3509 . 7	27—28	3188 . 6	42—43	2650 . 5	57—58	1931 . 5		
13	14	3495 . 6	28—29	3159 . 3	43—44	2607 . 6	58—59	1878 . 3		
14	15	3480 . 4	29—30	3128 . 8	44—45	2564 . 1	59—60	1824 . 6		
							60—61	1770 . 2		
							61—62	1715 . 4		

To

To render the application of the above table and ſubſequent proceſs more intelligible, I have annexed a ſketch of the projection and outline of the map of Germany.

HAVNIG drawn the contour of the country with all poſſible accuracy, and expreſſed it by a very fine line, obſerve what quadrilateral ſpaces lie entirely within it; encloſe theſe by a ſtrong black line, and the area of all within it we may call the *integral area*, as it conſiſts of entire quadrilaterals; and that without it, the *fractional area*, as it is compoſed of parts of quadrilaterals.

THE *integral area* may be found with the greateſt accuracy thus: I find, in the table, that a quadrilateral between the 53d and 54th parallel contains 2138.2 ſquare miles, and that there are ſix ſuch in the *integral area*; therefore $2138.2 \times 6 = 12829.2$ will give the true contents of that portion of it which lies between theſe parallels, and a ſimilar operation for the zones between the other parallels will give 144023.6 as the entire *integral area* of Germany.

THUS far (ſuppoſing the map correct) there is no poſſibility of error. The contents of the fractional area can be found only by approximation: The method I propoſe is this:

LET it be required to find the area of the ſpace A in the quadrilateral a b c d: having conſtructed a ſcale, from the extent of a degree of latitude, reduce the irregular waving contour *a b c* *Fig.* 1.

into a rectilineal outline, by ſhort balance, or (as they are vulgarly called) give and take lines, drawn by the aſſiſtance of a tranſparent ruler, with a thin fiducial edge: The polygonal ſpace A muſt then be reduced to a triangle by an operation familiar to ſurveyors, and its area found, by the uſual mode of multiplying half its altitude into its baſe: By a ſimilar proceſs, find the area of the polygonal ſpace B, which added to that of A, will give the area of the quadrilateral; and if this nearly coincides with the ſum, which reſults from multiplying its mean extent in longitude by 60, there has been no material error in the operation: The ſum of the areas thus found will be frequently very different from the true area of the quadrilateral, ſhewn by the table, and the error muſt be corrected by dividing the difference between the true and meaſured areas proportionally between the ſpaces A and B, thus,

LET the quadrilateral a b c d be ſituate between the 50th and 51ſt parallels of latitude, and its true area of courſe 2287 ſquare miles: But, in conſequence of the inevitable deviation from truth, in the projection of the map, this area produces by menſuration only 2247 ſquare miles; the ſpace A 1032, and the ſpace B 1215: To find the true areas of A and B,

as 2247 the meaſured area of the quadrilateral.
: 1032 the meaſured area of A.
:: 2287 the true area of the quadrilateral.
: 1050 the true area of A.

as

as 2247 the meaſured area of the quadrilateral.
: 1215 the meaſured area of B.
:: 2287 the true area of the quadrilateral.
: 1237 the true area of B.

When the true extent of the portions into which the contour line divides the quadrilaterals is thus adjuſted, the addition of the ſeveral numbers in the *fractional area* to the *integral* will give the contents of the map: and to prevent any confuſion in the numbers to be added, it will be adviſeable to ſtain the *fractional area* one uniform colour, and to add the numbers in the different zones in ſeparate columns.

By the above rules I meaſured the annexed map of Germany, including the Auſtrian Netherlands, taken from Mr. D'Anville: The following table ſhews the reſult and its relative proportion to the areas of different authors.

		Difference between each and D'Anville's map.
Mr. D'Anville's map taken from his firſt part of Europe, 1754 -	192552	
Templeman's Survey of the Globe -	156950—35602 —	
Buſching, 1780 - -	177984—14568 —	
(Statiſtiche Ueberſicht) in German, 1786 - - -	192000— 552 —	
Crome on the Size and Population of Europe, 1785 - - -	204736—12184+	

If the above method of finding the *fractional area* is not thought ſufficiently correct, I would propoſe the following, more laborious, indeed, but, if properly executed, more accurate.

A Quadrilateral, not exceeding one degree in extent, when accurately drawn on a plane ſurface, will produce an area by careful menſuration differing about 1. 3 ſquare miles in 1000 from its true extent on the ſphere; this error therefore being immaterial, delineate each quadrilateral in the fractional area on a ſeparate paper, and having drawn the contour line through it with great care and accuracy, meaſure each part as directed above, and if the ſum of the areas reſulting from this operation nearly equal that in the table, the operation may be conſidered as free from material errors; the ſmall difference that ariſes muſt be proportionally divided between the parts of the quadrilateral, that the whole and parts may be perfectly conſiſtent, and the numbers thus corrected written in their proper ſpaces in the map.

Area

Area of Germany and Switzerland, according to the Map of Mr. D'Anville, meaſured by the foregoing Method.

GERMANY.

	Fractional Areas.
Between the Parallels 45 and 46	530 . 0
	112 . 0
	80 . 0
	390 . 0
	1242 . 0
	380 . 0
	2734 . 0
46—47	127 . 0
	2279 . 6
	1672 . 6
	1094 . 0
	2236 . 6
	1792 . 6
	30 . 0
	9232 . 4
47—48	581 . 0
	658 . 0
	1738 . 6
	380 . 0
	6 . 0
	3363 . 6
48—49	960 . 0
	2198 . 1
	580 . 0
	10 . 0
	3748 . 1
49—50	5 . 0
	30 . 0
	230 . 0
	1050 . 0
	1105 . 0
	20 . 0
	2044 . 7
	1480 . 7
	25 . 0
	5990 . 4

	Fractional Areas.
50—51	400 . 0
	1443 . 8
	1872 . 8
	23 . 0
	3739 . 6
51—52	400 . 0
	570 . 0
	1007 . 0
	1129 . 8
	1426 . 8
	961 . 0
	65 . 0
	5559 . 6
52—53	825 . 0
	1082 . 0
	1907 . 0
53—54	270 . 0
	1338 . 2
	1928 . 2
	2014 . 2
	640 . 0
	20 . 0
	6210 . 6
54—55	275 . 0
	848 . 0
	896 . 0
	63 . 0
	41 . 0
	130 . 0
	824 . 0
	520 . 0
	30 . 0
	270 . 0
	867 . 0
	907 . 6
	Fermeren I. 50 . 0
	12 . 0
	Rugen I. 310 . 0
	6043 . 6

GERMANY.

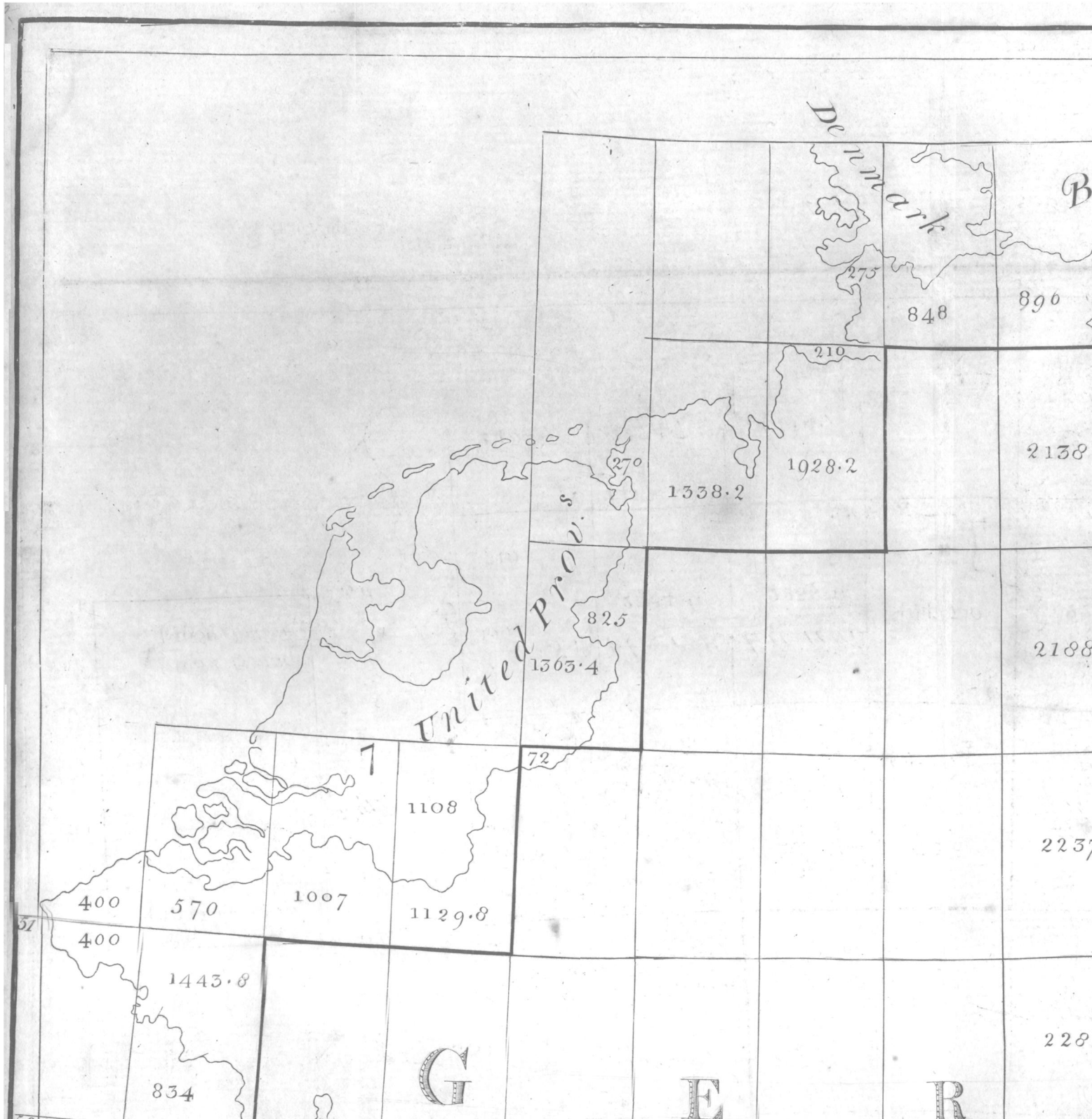
Between 45 and 46 - -	2734 . 0
46 — 47 - -	9232 . 4
47 — 48 - -	3363 . 6
48 — 49 - -	3748 . 1
49 — 50 - -	5990 . 4
50 — 51 - -	3739 . 6
51 — 52 - -	5559 . 6
52 — 53 - -	1907 . 0
53 — 54 - -	6210 . 6
54 — 55 - -	6043 . 6
Total *fractional area* -	48528 . 9
Total *integral area* -	144023 . 6
Total area of Germany -	192552 . 5

Germany, in the above area, comprehends the Auſtrian Netherlands, Dutch Limburg, Pruſſian Guelders, that part of Iſtria ſubject to the Houſe of Auſtria, and the territory of Aquileia.

SWITZERLAND.

Between 45 and 46 -	85
	245
	105
46 — 47 -	73
	1594 . 6
	2384 . 6
	2289 . 6
	1947 . 6
	15 . 0
	70 . 0
47 — 48 -	460
	1497 . 6
	1770 . 6
	690 . 0
Total area of Switzerland -	13227 . 6

Switzerland, in the above are, comprehends the Swiſs-cantons, their allies, their ſubjects, and the ſubjects of their allies.


Denmark
B
275
848
896
210
270
1338·2
1928·2
2138
United Prov.s
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400
570
1007
1129·8
51
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834
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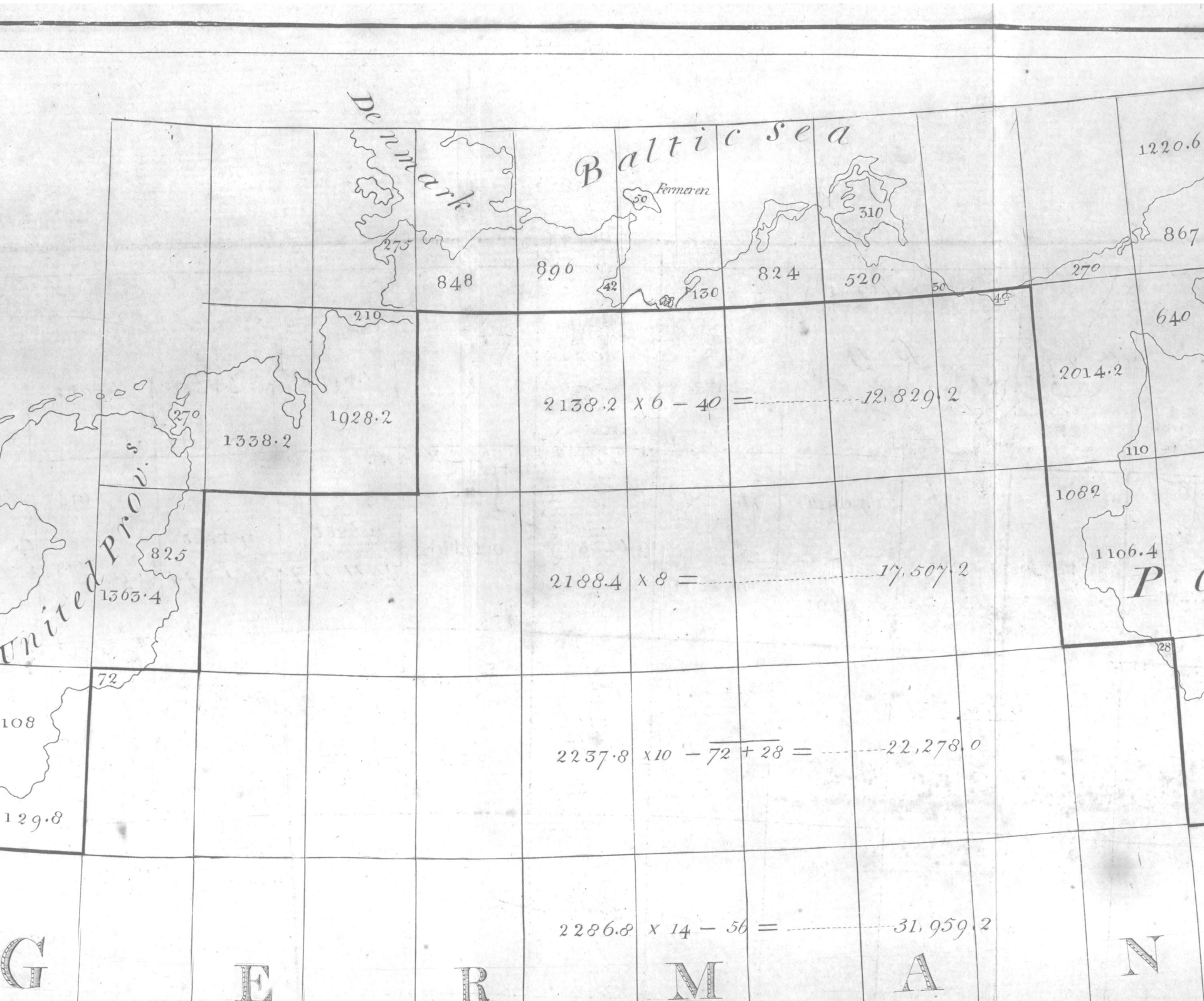
Denmark
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1220.6
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2138.2 x 6 − 40 = 12,829.2
270
1338.2
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110
1082
United Prov.s
825
1363.4
2188.4 x 8 = 17,507.2
1106.4
P
28
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2237.8 x 10 − 72 + 28 = 22,278.0
129.8
2286.8 x 14 − 56 = 31,959.2
G
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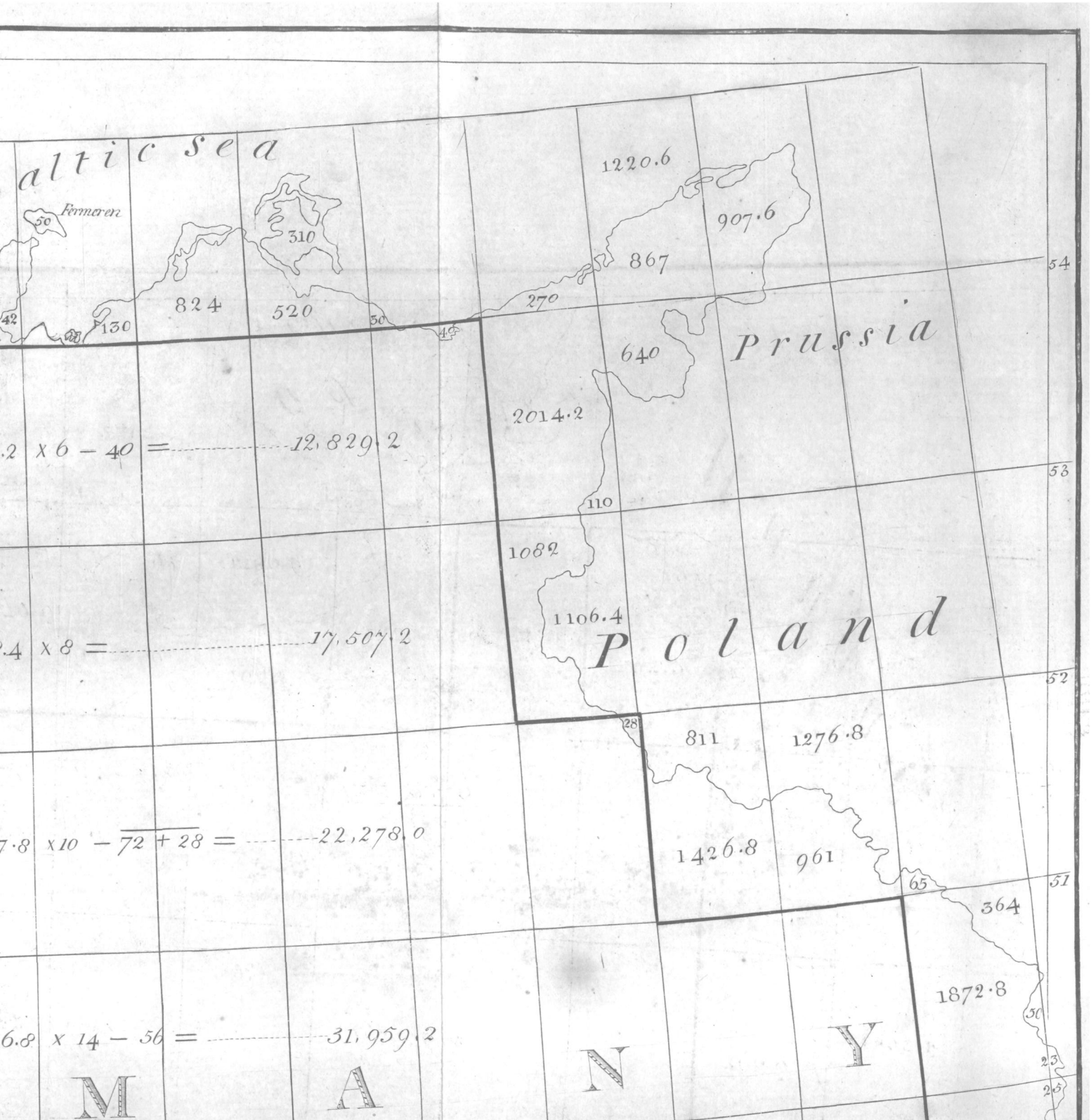


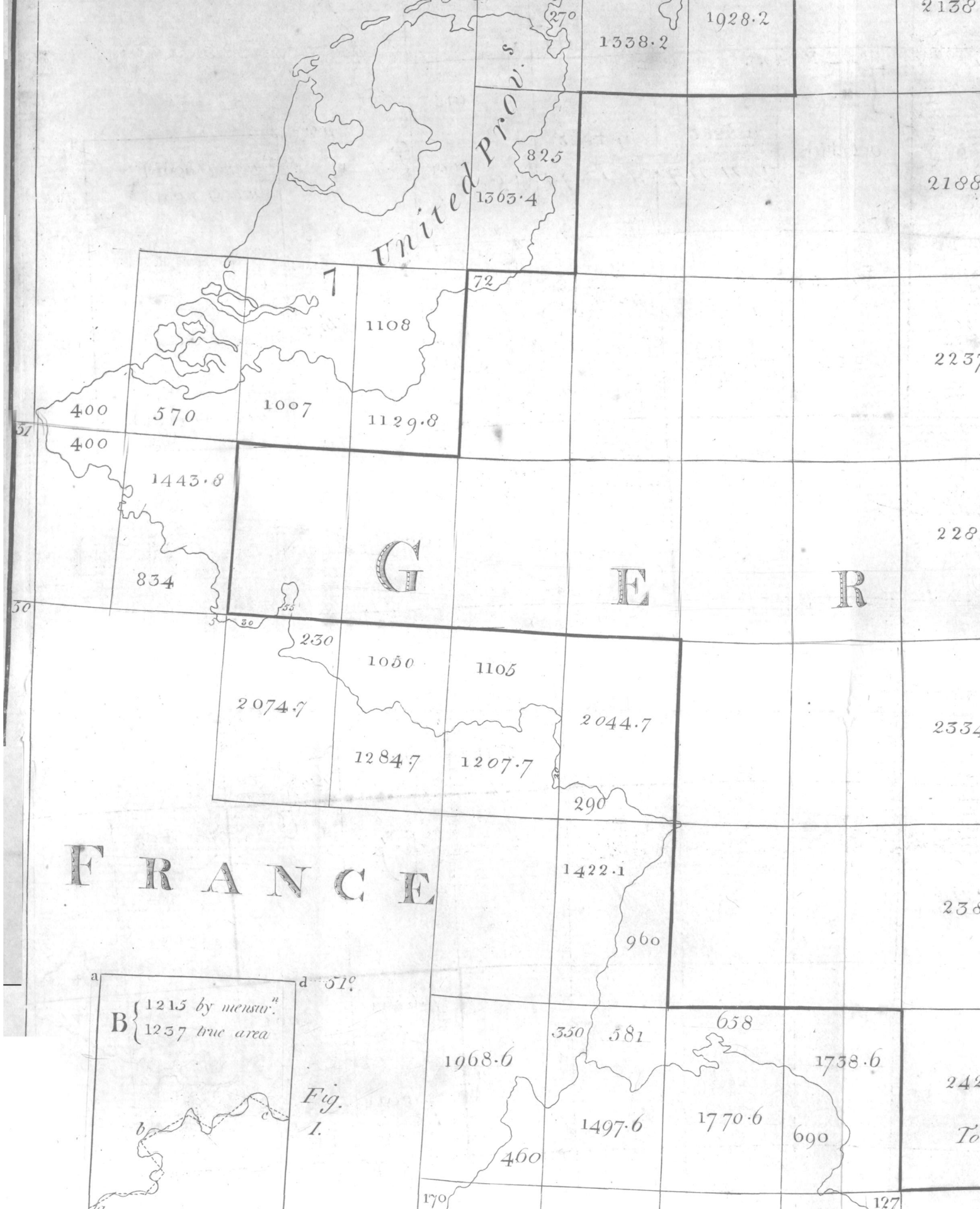
.2 x 6 − 40 = ... 12,829.2

.4 x 8 = ... 17,507.2

7.8 x 10 − 72 + 28 = ... 22,278.0

6.8 x 14 − 56 = ... 31,959.2

M A N Y



7 United Prov.s
G E R
FRANCE
270
1338·2
1928·2
825
1363·4
72
1108
1007
1129·8
400
570
400
1443·8
834
51
50
55
30
230
1050
1105
2074·7
2044·7
1284·7
1207·7
290
1422·1
960
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658
1968·6
1738·6
1497·6
1770·6
690
460
170
127
a
d 51°
B { 1215 by mensur.n
1237 true area
b
c
Fig. 1.

270
1338.2
1928.2
2138.2 x 6 − 40 = 12,829.2
110
1082
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P
United Prov.s
825
1363.4
2188.4 x 8 = 17,507.2
28
72
108
129.8
2237.8 x 10 − 72 + 28 = 22,278.0
2286.8 x 14 − 56 = 31,959.2
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2382.1 x 8 19,056.8
350
581
658
1968.6
1738.6
1497.6
1770.6
690
460
170
127
2428.6 x 6 = 14,571.6
2,474.6
Total of the Integral Area 144,023.6

.2 x 6 − 40 = ... 12,829·2

.4 x 8 = ... 17,507·2

7·8 x 10 − 72 + 28 = ... 22,278·0

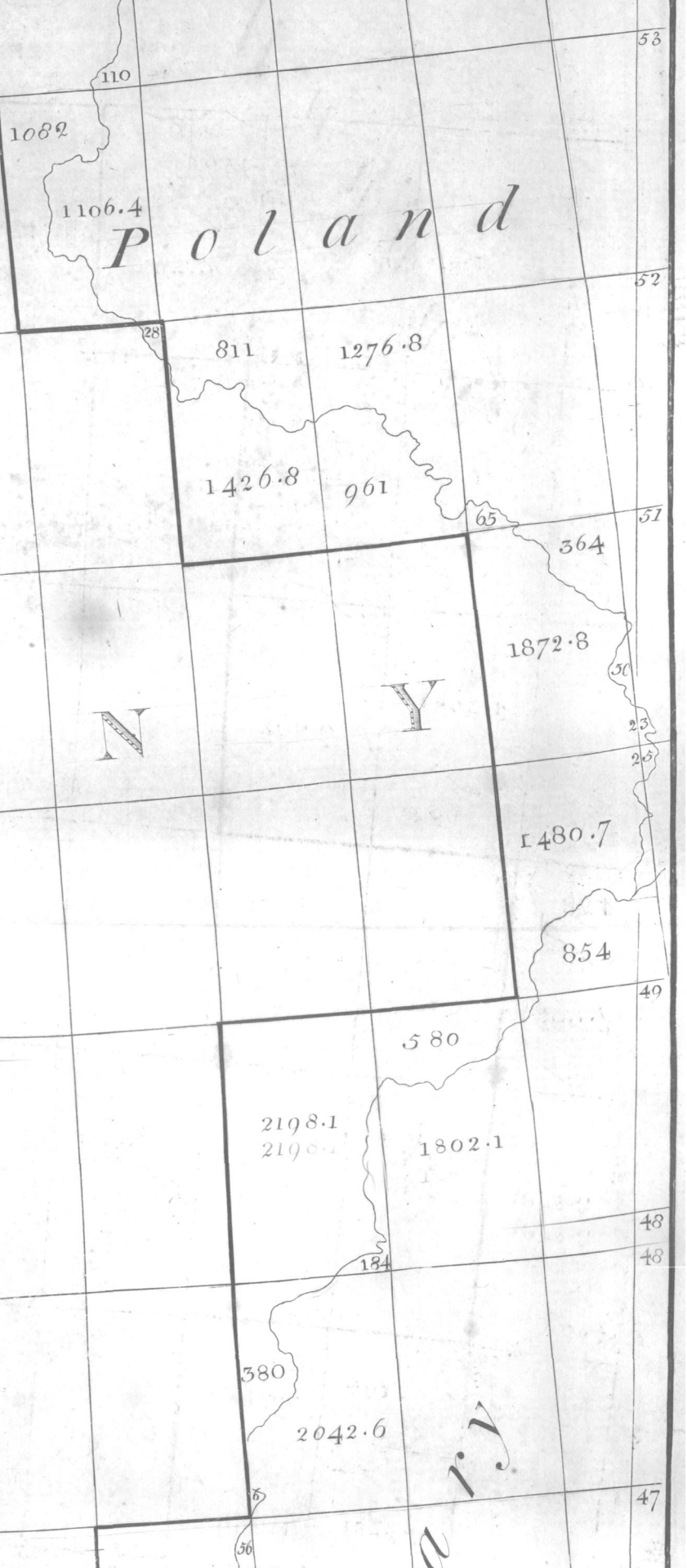
6·8 x 14 − 56 = ... 31,959·2

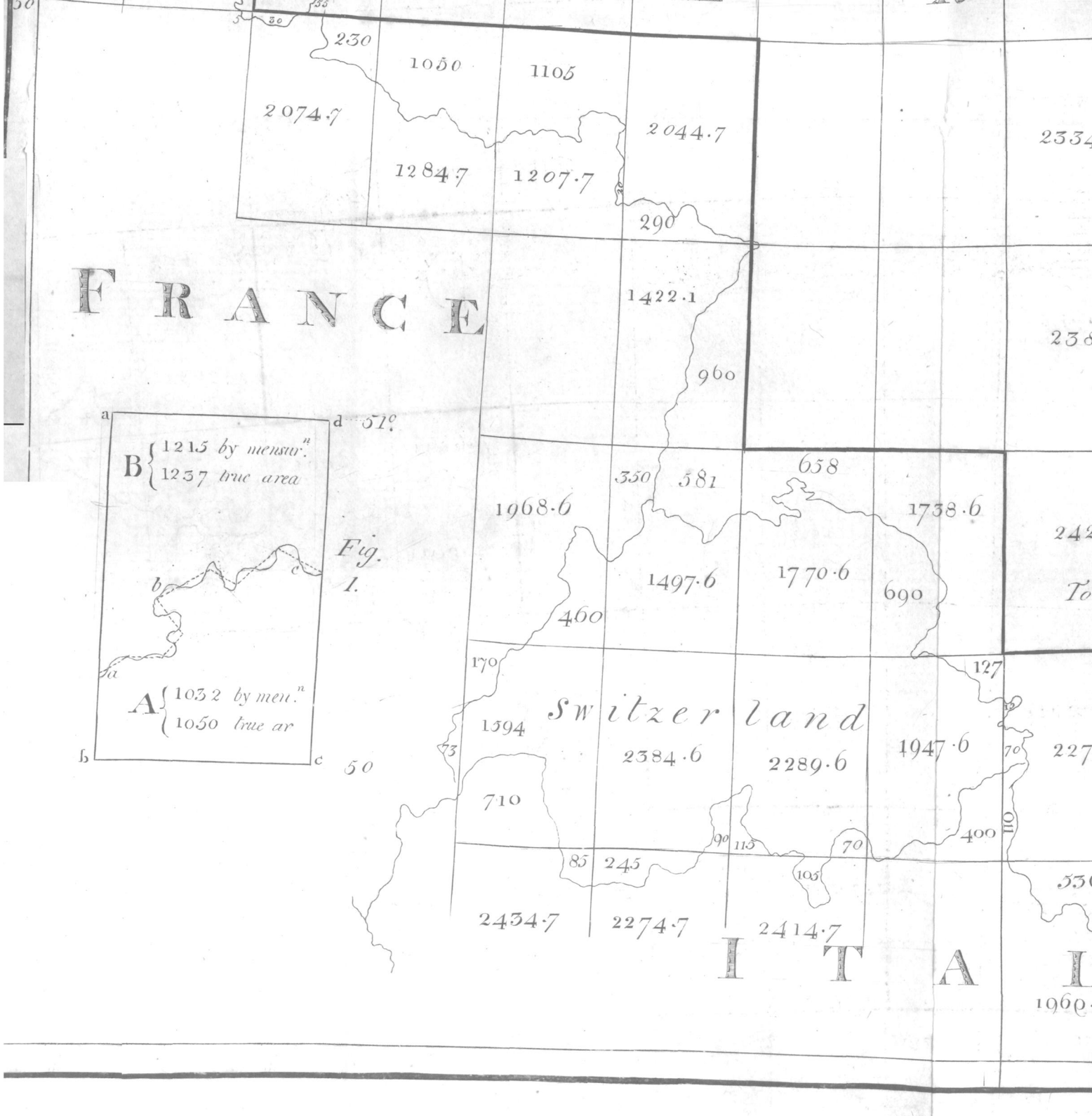
4·7 x 10 ..... 23,347·0

2·1 x 8 ..... 19,056·8

286 x 6 = ... 14,571·6
2,474·6
144,023·6

tal of the Integral Area




FRANCE
Switzerland
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B { 1215 by mensur.n
1237 true area
A { 1032 by mens.n
1050 true ar
a
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d
Fig. I.
230
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1284.7
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290
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658
350
581
1968.6
1738.6
1497.6
1770.6
690
460
170
127
1594
2384.6
2289.6
1947.6
73
70
710
400
110
90
115
70
85
245
105
2434.7
2274.7
2414.7

1050
1105
2044.7
2334.7 x 10
23,347.0
284.7
1207.7
290
1422.1
E
2382.1 x 8
19,056.8
960
350
581
658
1968.6
1738.6
24286 x 6 =
14,571.6
2,474.6
144,023.6
Total of the Integral Area
1497.6
1770.6
690
460
170
127
1094
1672.6
1792
Switzerland
1594
2384.6
2289.6
1947.6
2279.6
2236.6
2474.6
73
70
710
802
1380.6
90
115
70
400
110
85
245
105
530
112
80
390
1242
380
2434.7
2274.7
2414.7
2407.7
Ter. of Aquileia
istria
ITALY
G. of
Venice
1960.7

1480·7
854
49
580
2198·1
1802·1
48
184
380
2042·6
47
56
30
1672·6
1094
2236·6
2474·6
1792·6
626
802
1380·6
46
112
80
390
1242
380
2407·7
Y
Ter. of Aquileia
G. of
istria
Venice
45
Hungary